SPE 108762

# Modeling a Finite-Length Sucker Rod Using the Semi-Infinite Wave Equation and a Proof to Gibbs' Conjecture

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This paper was prepared for presentation at the 2007 SPE Annual Technical Conference and Exhibition held in Anaheim, California, U.S.A., 11–14 November 2007.

## Abstract

In this paper, we study semi-infinite spatial domain wave equations modeling the real world problem of longitudinal waves propagating along a finite length long slender homogeneous elastic rod. To conclude, we provide a rigorous, yet elegant proof to Gibbs' Theorem and illustrate its validity with an existing well.

## Introduction

In the landmark patent and paper by S.G. Gibbs[1,2] new methods are presented for diagnosing and predicting the behavior of sucker-rod pumping systems, respectively. In Gibbs[2] the method employs two boundary conditions. Since it is a prediction method, the two boundary conditions are specified at both the polished rod and at the pump. The first boundary condition is the surface position of the polished rod, while the second is a Robin boundary condition (i.e., it involves position and load at the pump) which simulates the downhole pump condition.

However, it is the solution in Gibbs[1] which is the subject of this paper. The result uses a separation of variables technique to solve the viscous damped wave equation on a *semi-infinite domain* and utilizes the measured surface position and load as the two necessary boundary conditions to model a rod string of *finite* length. Therefore, there is no explicit information about the position or load at the pump. In light of this fact, how are the reflections from the end of the rod string accounted for and measured in this model of a semi-infinite rod string? Admitting a clever revelation, it was realized that clandestinely embedded in these measured surface position and load boundary conditions are the reflected tension and compression waves that travel back and forth along the rod string from the pump to the polished rod and back. Utilizing this discovery and employing the semi-infinite model, with the boundary conditions of the pump embedded in the boundary conditions of the polished rod, the solution to the semi-infinite viscous damped wave equation is found to be exactly the solution to the model of the viscous damped wave equation of a finite length rod string. Thus, using the semi-infinite model, we can correctly model the wave propagation and reflection along a finite length rod string.

## Gibbs' Historical Foreword/Preamble

The dynamometer has been used to manage and troubleshoot rod pumping wells for over 80 years. Its original application was to determine loading on surface equipment namely the structure, reducer and prime mover. A typical measurement, called a dynamometer card, is a plot of rod position versus rod load such as shown in Figure 1. Owing to the elasticity of the rod string and its data transmission capability, information about downhole operating conditions travels to the surface in the form of stress waves. Recognizing this, rod pumping technicians of the past began to use the shape of the dynamometer card to qualitatively infer downhole conditions, particularly at the pump. A typical goal could be to determine if the pump is filling completely with liquid or if free gas is entering the pump to the detriment of pump efficiency. These visual determinations were not always definitive and were highly dependent upon the skill and experience of the analyst. A difficulty arises because the finite velocity of the stress waves traveling up and down in the rod string creates a time delay in surface manifestation of events occurring at the downhole pump and vice versa. This coupled with the flexibility of the rod string makes determination of downhole operating conditions difficult to visually discern from surface measurements with a high degree of accuracy and certainty.

In 1936 W.E. Gilbert and S.B. Sargent revealed an instrument which literally measured a subsurface pump dynamometer card. It was a mechanical device which was run just above the pump in the rod string. It allowed a small number of pump dynamometer cards to be collected before being recovered by pulling the rods. It scribed the pump card on a rotating tube whose angular position was made proportional to plunger position with respect to the tubing. Pump load was measured as proportional to the stretch of a calibrated rod within the instrument. Since the sucker rod string had to be pulled to recover the pump cards, the instrument was costly to use. But it provided valuable information relating the shape of the pump cards to various operating conditions known to exist in pumping wells (full fillage, gas interference, fluid pound, pump malfunctions,