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A New Method of Evaluating the Productivity Index for Nonlinear Flows

E. Aulisia and A. Ibragimov, Texas Tech. University, and P.P. Valkó, SPE, and J.R. Walton, Texas A&M University

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Abstract

This paper addresses the effects of non-linear, non-Darcy flows in reservoir on the value of the productivity index. The productivity index (PI) of the well draining a reservoir with no flux on the exterior boundaries is defined as the ratio between the production rate and the pressure drawdown. Experience shows that during the dynamical process of hydrocarbon recovery this ratio stabilizes to some constant value, which, in general, is a non-linear function of both the pressure drawdown and the production rate.

The linear (Darcy) case is well understood, and excellent approximate formulae are available to compute the PI for various well-reservoir geometries. These formulae are generally obtained through a semi-analytical solution of the transient problem. To handle the more realistic non-linear situation, the current practice is to solve first the nonlinear problem many times for different values of production rate, and then to add some ad-hoc corrective parameter(s) in the linear formulae in order to reproduce the non-linear nature of the flow.

Our approach, based on recent progress in the modeling of transient Forchheimer flows, uses partly non-numerical techniques to evaluate the productivity index. It provides, for a wide class of reservoir geometries, an accurate enough approximate algebraic relation between the PI for the non-linear Darcy-Forchheimer flows and the PI for the Darcy flows.

We show that the solution of the original problem can be obtained by applying direct variational methods, which are computationally less expensive than the grid based techniques. In addition, by using the existing and newly developed approaches in the theory of symmetrization, alternative "non-analytical" algorithms are presented to assist optimal well design, without repeatedly solving numerically the reservoir simulation problem.

The approach will be demonstrated on practical well optimization problems.

Introduction

To clarify the main results of this work we will review the definition of the productivity index (PI) and underline some assumptions about non-linear flows. Formally speaking, the productivity index is one of the most basic characteristic of the well performance and does not require any assumption about the equations of the flow motion and the equation of state of the fluid. The concept of productivity index expresses the following fact: once the well production is, in some sense, stabilized, then the ratio between the production rate and the pressure drawdown (difference between the reservoir average pressure and the well average pressure) is practically independent from the production history or even from the operating conditions. The higher is the values of the PI the better are the performances of the reservoir.

Consider a bounded and isolated reservoir with no-flow on outer boundary. A well producing with either constant rate or constant pressure is characterized by the productivity index defined in (Muskat, 1937; Larsen, 2001; Dietz, 1965; Dake, 1978 and Raghavan, 1991) as

$$PI(t) = \frac{Q(t)}{\Delta P(t)}.$$

Here $Q(t)$ is the production rate and $\Delta P(t) = P_a(t) - P_w(t)$ is the pressure drawdown, where $P_a(t)$ is the average reservoir pressure and $P_w(t)$ is the flowing bottom-hole pressure. We are particularly interested in the asymptotic (late time) value of the PI. For constant production rate, $Q(t) = Q$ stabilization means that the difference between the average and the well-bore pressure (the denominator) becomes time invariant. This flow regime is called pseudo-steady state (PSS). In the case of constant wellbore pressure, both the numerator and denominator change in time, but their ratio asymptotically stabilizes to a constant value, leading to the flow regime called boundary-dominated (BD) (see Muskat, 1937; Larsen, 2001; Dietz, 1965; Dake, 1978; Raghavan, 1991; and references therein).

In case of linear Darcy flow for slightly compressible fluid, the most popular way to evaluate the productivity index for constant thickness reservoir is based on the representation of the PI as the following product $PI = F_0 J$. Here $F_0 = \frac{2\pi k h}{B\mu}$ depends only on the fluid and on the porous media properties together with the reservoir thickness. J is the dimensionless productivity index which depends on the well-reservoir geometry and on the type of flow regime. The PSS