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Numerical Inversion of Laplace Transforms in the Solution of Transient Flow Problems with Discontinuities

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Abstract

Laplace transformation provides advantages in the solution of many pressure-transient analysis problems. Usually, these applications lead to a solution that needs to be inverted numerically to the real-time domain. The algorithm presented by Stehfest in 1970 is the most common tool in petroleum engineering for the numerical inversion of Laplace transforms. This algorithm, however, is only applicable to continuous functions and this limitation precludes its use for a wide variety of problems of practical interest. Other algorithms have also been used, but with limited success or popularity. A recent algorithm presented by Iseger in 2006 removes the restriction of continuity and provides opportunities for many practical applications. This paper exploits the useful features of the Iseger's algorithm in the inversion of continuous as well as singular and discontinuous functions that arise in the solution of pressure-transient analysis problems. The most remarkable applications are in the problems that require the use of piecewise continuous and piecewise differentiable functions, such as the use of tabulated data in the Laplace transform domain, deconvolution algorithms, and solutions that include step-rate changes as in the mini-DST tests.

Introduction

Since the seminal work of van Everdingen and Hurst (1953), Laplace transformation has been a standard tool for transient fluid-flow problems in porous media. Laplace transformation is mostly used for the solution of initial-boundary-value problems and in the evaluation of integral equations of convolution type. For some of these applications, an exact (analytical) inversion is not possible and numerical inversion is the only resort. For some others, numerical inversion is chosen also because of convenience.

Kryzhniy (2006) defines three general cases encountered in the inversion of Laplace transforms:

1. The solution is obtained in closed form analytically in the Laplace domain: The inverse transform can be found analytically or with the help of tables. Numerical inversion may also be used if the solution is too complicated.
2. The Laplace transform of the solution is computable in the complex half-plane of convergence: In this case, the inverse Laplace transform may be obtained by the evaluation of the complex integral of inverse transformation. Numerical inversion based on the calculation of the Bromwich integral (when possible) yields good results.
3. The Laplace transform of the solution is computable or measurable on the real and positive axis only: In this case, there is no exact inversion formula. All numerical inversion methods encounter difficulties because the inversion problem is severely ill-posed.

Kryzhniy (2004) points out that a large group of exponential analysis problems encountered in transient problems, which involve real-valued Laplace transforms, are subject to inversion problems because no exact inversion formula is available for real-valued functions.

The focus of this paper is the numerical inversion of Laplace transforms in the solution of transient-flow problems with piecewise-differentiable data. These problems usually arise due to piecewise approximations to tabulated rates and pressures, interruption of production by shut-ins, and step-rate sequences as in mini-DST applications. In some of these cases, the function to be inverted may be continuous but not continuously differentiable (e.g., piecewise approximations to sampled data). In others, the function may be only piecewise continuous and piecewise differentiable (e.g., a sequence of drawdown and buildup periods). Laplace transformation of piecewise continuous functions include exponential contributions at the boundaries of its sections, which cause problems in numerical inversion. If the function is discontinuous at section boundaries, then most numerical inversion algorithms require special treatments or fail altogether. These piecewise