Uncertainty and the Volumetric Equation

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Jim Murtha *has been a consultant since 1992 focusing on building models that address uncertainty and training people to use them. He previously taught mathematics and petroleum engineering at Marietta College, where he also served as academic dean. He is a Distinguished Member of SPE, served as Distinguished Lecturer in Risk and Decision Analysis, and is principal author of the chapter on risk and decision analysis in SPE's* Petroleum Engineering Handbook. *He has also written a book on risk analysis and coauthored two texts in mathematics and statistics. Murtha earned a PhD degree in mathematics from the University of Wisconsin, an MS degree in petroleum and natural gas engineering from Penn State University, and a BS degree in mathematics from Marietta College.*

Jim Ross *is a consultant providing training and advisory services in reserve/resource classification and reporting and in unitization and redetermination issues. He has served on a wide range of SPE committees, including the Oil and Gas Reserves Committee, where he played the lead role in the development of the SPE/World Petroleum Council/American Association of Petroleum Geologists Resource Definitions and Classification System. He has received the SPE Management and Information Award, and is currently involved with the preparation of further SPE documentation intended to facilitate consistent resource classification. He is also a Bureau member of the United Nations' Ad Hoc Group of Experts on the Harmonization of Fossil Energy and Mineral Resources Terminology. Ross earned PhD and BSc degrees in geophysics from the University of Liverpool, and an LLM degree in petroleum law and policy from the University of Dundee.*

Every few years, the topic of correct inputs for use in the volumetric equation re-emerges. Our first recollection is from the 1997 SPE Hydrocarbon Evaluation and Economics Symposium when a dozen or so attendees remained after a session to discuss a logical flaw in a technical paper. All of those present agreed that they had been doing volumetric estimates incorrectly, due to their choice of distributions, and vowed to correct the situation. More recently, in the spring and summer of 2007, SPE's Reserves and Economics Technical Interest Group had a lively discussion on the same topic that ended without resolution and with extensive misunderstanding, in part because of perceived repetition of arguments. Now with the recent release of new reserves definitions and disclosure rules from the US Securities and Exchange Commission and comparisons with the Petroleum Resources Management System developed by SPE and other organizations, we may see more companies using probabilistic methods to estimate hydrocarbons in place, resources, and reserves, so we believe that it would be useful to reiterate the conclusion of those present at that 1997 meeting in Dallas.

The crux of the issue is: What is the correct mathematical basis for establishing appropriate uncertainty distributions as inputs to a probabilistic analysis of the volumetric equation? More specifically: Should the input distribution for each reservoir parameter be based on either

1. The range of values seen in the well data for that parameter, or,
2. The range of uncertainty in the reservoir average for that parameter?

A Simple Model

For illustrative purposes, consider using the volumetric equation in the form: oil initially in place (OIIP)=$7758Ah\phi(1-S_w)/B_o$. The following discussion would also apply to other forms of the equation, such as using gross rock volume and net-to-gross ratio, for example.

The total area of the reservoir, *A*, is 1,920 acres and 24 wells have been drilled on the basis of 80 acre spacing. The net pay, *h*, varies between 11 and 165 ft in the wells drilled to-date **(Table 1)** and statistical analysis of the net pay in the 24 wells shows that the average (mean) net pay is 81.5 ft. For the full calculation of OIIP, we would also need to consider uncertainties in area, porosity, water saturation, and the oil formation volume factor, but we will focus on net pay here.

TABLE 1—WELL RESULTS FOR NET PAY

Well	*h* (ft)	Well	*h* (ft)	Well	*h* (ft)	Statistics	
1	42	9	112	17	102	Sum	1,956 ft
2	71	10	120	18	165	Count	24
3	68	11	11	19	72	Mean	81.5 ft
4	129	12	106	20	68	SD*	40
5	54	13	63	21	101	SEM**	8.16
6	48	14	61	22	32	Max.	165 ft
7	111	15	145	23	54	Min.	11 ft
8	54	16	134	24	33		

*Standard deviation (SD)
**Standard error of the mean (SEM) = SD/√(count)

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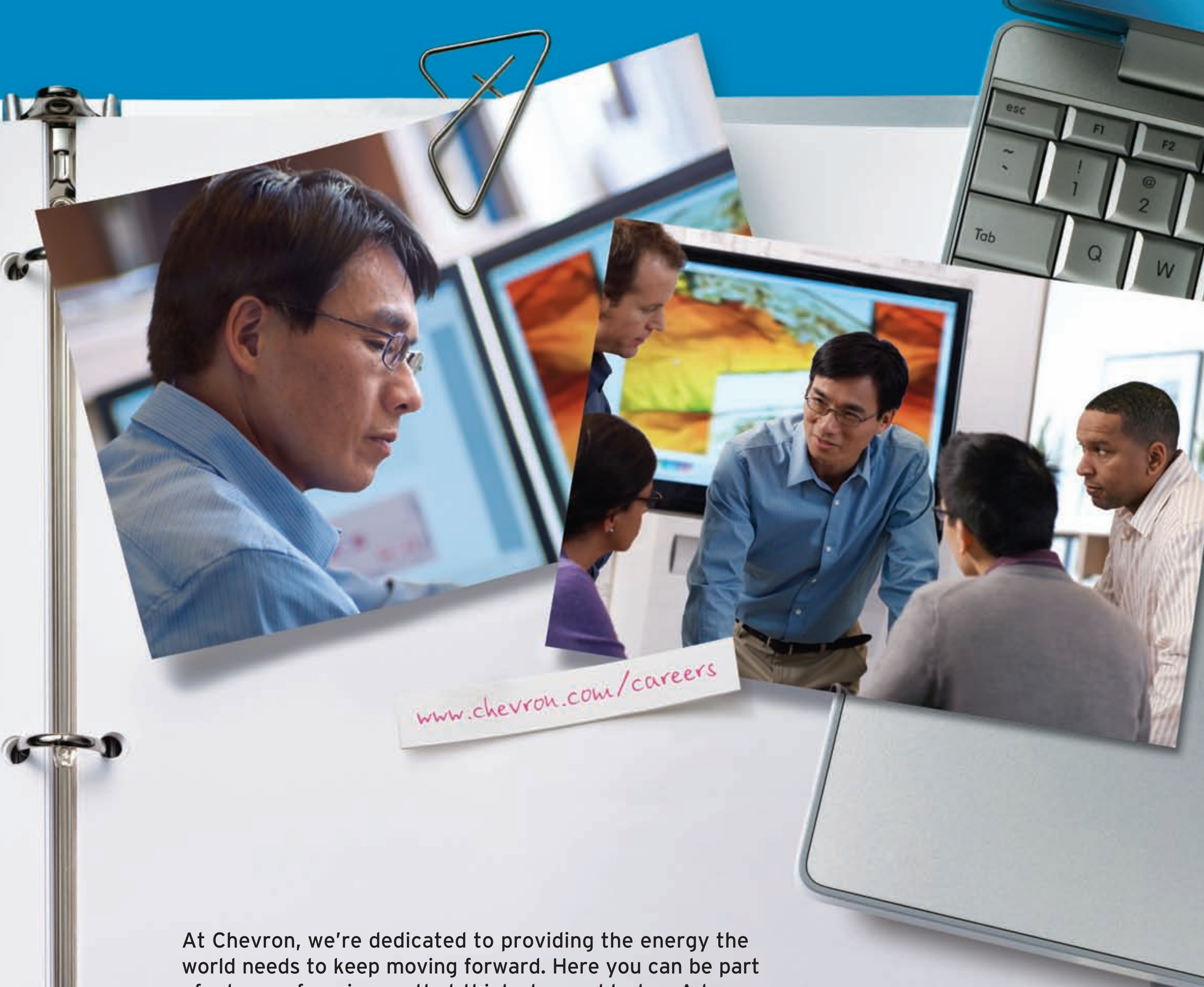

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Deterministic Analysis

If we wished to make a determination of the best estimate of the total OIIP in this reservoir, we can determine the bulk (net pay) volume from the product of the area (1,920 acres) and the average net pay (81.5 ft), which is 156,480 acre-ft. This observation should be obvious: volume is computed by multiplying the area by the average thickness.

Probabilistic Analysis

Probabilistic output from the equation

$$OIIP=7758Ah\phi(1-S_w)/B_o$$

requires uncertainty distributions for each of the five input parameters (noting that sometimes gross rock volume and net-to-gross ratio are used as inputs instead of A and h). In our simple example, we are focusing on net pay, h, and would like to derive an estimate of the uncertainty in bulk volume ($A\times h$). So we need to determine the correct distribution of h.

First, it is worth noting that Monte Carlo analysis involves a large number of iterations, each of which is a single *deterministic* calculation of OIIP using the volumetric equation. Nothing magic in this, but obviously each deterministic calculation performed within the Monte Carlo analysis must be based on realistic input data to generate a realistic estimate of OIIP.

The range of values for h is shown in Table 1, and has a minimum of 11 ft, a mean of 81.5 ft, and a maximum of 165 ft. We could "fit" a probability density function to these data points and that would reflect the range of values seen in the well data for that parameter (option 1 in our introduction). Alternatively, we could use a distribution based on the uncertainty in the mean value (option 2), which can be described by a normal distribution also with a mean 81.5 ft but with a SD (known as the SEM) of 8.16. This would be a much narrower distribution, with a total range (approximated by ±3 SDs) from 57 to 106 ft.

If we use these two distributions, we will find that option 1 would indicate a range of uncertainty in the bulk volume for the whole reservoir from less than 21,120 acre-ft (1920×11) to over 316,800 acre-ft (1920×165), while option 2 would lead to a range of uncertainty of 109,248 to 203,712 acre-ft. So which one is correct?

On the basis of option 1, for the bulk volume to be 21,120 acre-ft, the average net pay over the whole reservoir would have to be 11 ft. This is untenable given the results of the 24 wells, since every well bar one has found more than 11 ft of net pay and some wells have found much more. Similarly, it is evident that a value of 316,800 acre-ft would require an average net pay over the whole reservoir of 165 ft, which is also untenable.

Since the equation $A\times h$ requires the use of average h to correctly estimate the volume, it follows that a calculation of uncertainty in $A\times h$ must be based on an uncertainty distribution in average h. The same logic applies to the other reservoir parameters used in the volumetric equation.

More Data

As a reality check, consider infill drilling to 20 acre spacing in the same reservoir. We now have 96 well data points for the reservoir. Since the infill wells would generally be expected to find net pay values that are somewhere in the range of values seen in the adjacent wells, the overall distribution of net pay, including its mean and SD, will not change by much. However, our new estimate of the mean should be a "better" estimate of the actual average net pay over the reservoir, i.e. have less uncertainty, and this will be reflected in a smaller value for the SEM.

Imagine now that we have 10,000 wells in this reservoir. The distribution of net pay values (option 1) will still look roughly the same, but the range of uncertainty in our estimate of the mean (option 2), and hence the uncertainty in our estimate of OIIP, must have improved significantly.

Less Data

This is where it gets trickier and, as is usual in our business, requires some careful judgment. The problem, of course, is the lack of a statistically valid sample, but what has not changed is the theoretical basis for the input parameters to the volumetric equation. We still need an estimate of the mean for each parameter and a distribution of our estimated uncertainty in that mean.

Let's say that we have only drilled the first three wells, all drilled in a poorer part of the reservoir and with net pay values of 42, 71, and 68 ft, respectively. Where we had a lot of data, using the sample distribution (as reflected in the SD of the well values) instead of the distribution of uncertainty in the mean (based on the SEM) led to an overestimate of the range of uncertainty in the mean. With few data points, the opposite can happen and the range of uncertainty will be underestimated. The three wells have an average of 60.3 ft and, by chance, a relatively narrow range from 42 to 71 ft. We know from the results of the 24 wells that the average net pay for this reservoir is actually outside this range, at 81.5 ft. The key point is that using the sample distribution (of these three wells) is just as invalid in this situation as it was where we had a lot of data.

What we would need to do is to look at any regional data and consider the environment of deposition of the reservoir so that we could establish a realistic distribution of net pay over the reservoir, ideally based on contouring of trends, and then use that best-estimate geological model to compute a value for the average net pay. Then we should run sensitivity cases investigating alternative models and/or contouring options from which we can determine a range of possible values for average net pay (noting that there are software packages available that can do all the computations for us, based on some input constraints).

Conclusion

A probabilistic analysis of the volumetric equation in the form

$$OIIP=7758Ah\phi(1-S_w)/B_o$$

or in other comparable forms, requires distributions of uncertainty for each parameter which, apart from A (or gross rock volume), reflect the *average* value of that parameter over the area (or volume) for which OIIP is being calculated. In the above example, this is the entire reservoir. To be more precise, the requirement is for the average net pay over the area of the reservoir, the average porosity over the bulk volume, and the average oil saturation over the net pore volume. These computations are often dealt with through the application of 3D geological models.

Since the equation requires the use of averages for reservoir properties as input, it is the uncertainty in the reservoir average that must be the basis for a probabilistic analysis. It is mathematically incorrect to use distributions of the well data values and such an approach will lead to an invalid distribution of OIIP. **JPT**

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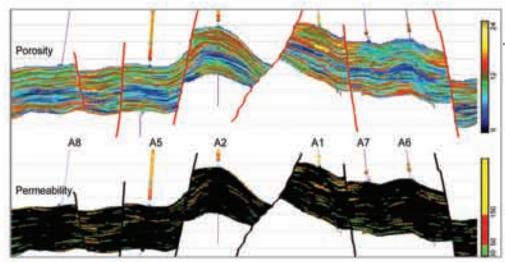

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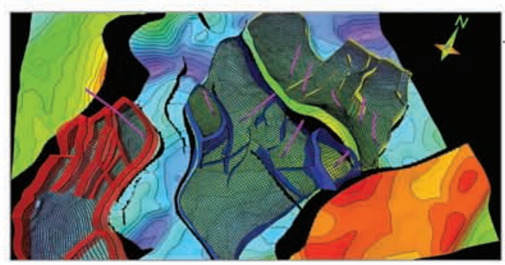

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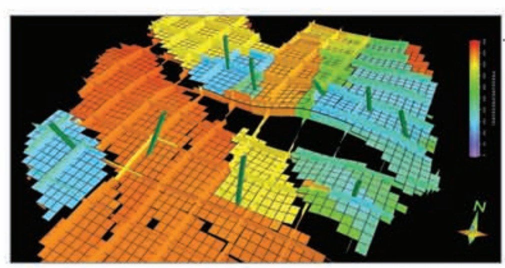

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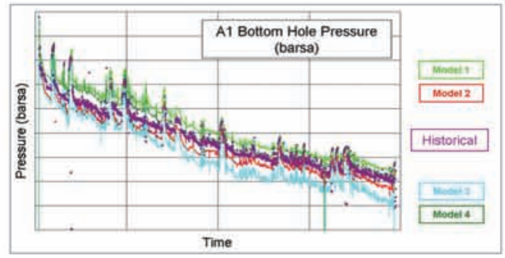

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