Abstract

Objective/Scope: Explicit time marching finite difference (FD) stencils have been broadly used for wave propagation simulation and comprise the most computational intensive part. Time marching step, which is determined by the stability condition and numerical dispersion, is a key factor for the computational cost. This paper is targeting to close the debate about the contradictory opinions on whether employing the Lax-Wendroff stencil permits larger time steps. Besides, we introduce a new algorithm for numerical dispersion analysis.

Methods, Procedures, Process: For the numerical stability condition, we analyze the stability condition using the von Neumann method for both second-order and Lax-Wendroff stencils and we also include a numerical test to confirm our derivations. In terms of the numerical dispersion problem, instead of the traditional used method to analyze the phase velocity errors in the frequency-wavenumber domain, we calculate the relative L2 norm of the waveform difference between the numerical solution and analytical solution which accumulates both phase and amplitude errors. We build new functions to determine the temporal grid size and the correctness of these functions are proved by numerical tests.

Results, Observations, Conclusions: Compared to the widely used second-order explicit time-marching FD stencil, the Lax-Wendroff stencil is much cheaper due to the larger temporal grid size required by the stability condition and numerical dispersion. In this paper, we proved that both theoretically and numerically, the Lax-Wendroff stencil enables $\sqrt{3}$ times larger time-step than a second-time-order stencil. This conclusion can close the debate about the contradictory opinions on whether employing the Lax-Wendroff stencil permits larger time steps. Based on comparing the relative L2 norm of the waveform difference, we build the model adaptive relations between the temporal grid size and number of wavelengths away from the source location. Based on these relations, we show that the Lax-Wendroff stencil is more than two times faster than the second-order stencil under the same error threshold, and the gain becomes larger with increasing offsets and higher frequencies which is more important for modern exploration applications. The equation we solve in this paper is acoustic wave equation, but similar analysis could be done for more complex equations.

Novel/Additive Information: The derived equations is new, model adaptive, and easily coded to be adapted by the current high performance computing seismic simulation applications (forward modeling, RTM, FWI, et, al.) in petroleum industry.