Pipeline Integrity Assessment Using Probabilistic Transformation Method and Corrosion Growth Modeling Through Gamma Distribution

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Summary
This paper presents a methodology to estimate the probability of failure of every individual corrosion defect and the residual likelihood of failure of an overall corroded pipeline segment after selected repairs, using structural probabilistic analysis based on the assumption of a stationary gamma distribution of the corrosion process. Metal loss size is taken as the load condition, and the allowable depth of the defect is taken as the resistance condition. The load condition is obtained applying the probabilistic transformation method (PTM) and the resistance condition is obtained through Monte Carlo simulation.

Using inspection data collected on a gas pipeline segment pigged two times, results of the suggested methodology are compared with those obtained by a classical approach based on a Gaussian distribution. It is determined that the use of Gaussian model overestimates for a short time horizon after inspection the failure probability, and suggests many useless repairs. The model developed in this paper instead authorizes a more rigorous estimation of failure probability and a significant reduction of maintenance costs.

Introduction
Probability of failure increases while pipelines become aged, and maintenance and rehabilitation strategies should be based on the prediction of the pipeline resistance behavior with time. Corrosion can be defined as a dynamic and complex destructive attack on pipelines. It is related to the physical properties of the pipeline, its surrounding environment, its mechanical properties, its coating, and the efficiency of its cathodic protection (Ahammed and Melchers 1996, 1997). The deterioration mechanism caused by corrosion is made possible once the pipeline is pigged several times. Nevertheless, because of the considerable length of pipelines carrying hydrocarbons, inspection by smart pigs is expensive, and its planning must take limited resources into consideration. Though the interval between two inspections is important, the longer that interval, the greater the uncertainty about the deterioration state of the pipeline. In this context, mathematical models are to be developed in order to serve as a way to optimize maintenance planning and to allow a quantitative evaluation of the remaining probability of failure during the interval between two inspections.

To estimate the failure probability of a corroded pipeline, corrosion rate is the fundamental parameter to be assessed. This evaluation is based on the comparison of the sizes of defects in successive ILI inspection runs (Worthingham et al. 2002). To make prediction of this deterioration possible, a minimum of two sequential inspections is theoretically required.

Structural Probabilistic Analysis and Quantitative Estimation of Failure Probability
Because of the influence of corrosion variability, resistance uncertainties, tool accuracy, and geometrical dimensions of the pipeline, many authors recommend the use of probabilistic analysis (Desjardins 2003; Alamilla et al. 2009). Deterioration-process evolution over time is uncertain, and it can be represented using a stochastic procedure. In this context, structural reliability approaches were applied from several years in many fields, in particular from nuclear and civil engineering.

Probabilistic structural analysis can be defined as the formulation of a mathematical model to calculate the probability of a structure to be in a specified state, taking into account randomness of its load and/or resistance properties, or the unknown values of same. The limit state is defined where the load and resistance conditions are equal. Once the load condition is greater than the resistance condition, failure occurs.

Considering that the load condition $S$ and resistance condition $R$ are, respectively, defined by the density of probability functions $f_S(s)$ and $f_R(r)$, the probability of failure $P_f$ is then given by

$$P_f = P(g = R - S \leq 0) = \int f_{g,k}(r,s) dr ds, \quad \text{ ..................(1)}$$

where $g$ is the performance function and $f_{g,k}(r,s)$ is the probability of density function.

Almost all of the quantitative approaches adopted use the load-resistant models under several assumptions of statistical distributions of operating pressure, maximum operating pressure, and corrosion rate. In the case of a corroded pipeline, it is judicious
to consider metal loss as the load condition and the allowable dimensions of the defect as the resistance (Younsi and Smati 2005). Otherwise, failure occurs once corrosion depth \( x \) reaches the critical depth \( x_{cr} \). Failure probability is then equal to the surface represented by the hatched area in Fig. 1, where \( Z \) represents the intersection point between load and resistance functions.

The main advantage of such representation lies in its simplification of the problem, which becomes a function of corrosion depth only. However, because of the dynamic nature of the deterioration process, the load is a time-related function and is increasing. By contrast, the resistance condition remains constant (Fig. 2).

Splitting the interval of time, separating two inspections, into \( N \) elementary intervals \( \Delta t \) and putting \( t_i=i\Delta t \), the probability of failure at \( t_i \) can be given as

\[
P_i(\tau_i) = P\left( g(\tau_i) = x_{cr} - X(\tau_i) \leq 0 \right) = \int_{\tau_i}^{\infty} f_{X_{\tau_i}}(x) \, dx + \int_{-\infty}^{\tau_i} f_{X_{\tau_i}}(x) \, dx. \tag{2}
\]

**Resistance Curve Modeling**

As a deterministic model, recommendations found in the works of Kiefner et al. (1973) and Kiefner and Vieth (1990) are frequently used. These works have the ASME B31G as a root (1991) (Appendix A).

Assuming that uncertainties in mechanical properties and geometrical parameters in Eq. A-3 are described using normal and log-normal distributions with a known mean and variance (Lee and Pyun 2002), the resistance distribution can be expressed through the Gaussian probability density function:

\[
f_{X_{\tau_i}}(x) = N\left( x / \mu_{x_{\tau_i}}, \sigma_{x_{\tau_i}} \right) = \frac{1}{\sigma_{x_{\tau_i}} \sqrt{2\pi}} \exp \left( -\frac{(x - \mu_{x_{\tau_i}})^2}{2\sigma_{x_{\tau_i}}^2} \right), \tag{3}
\]

where \( \mu_{x_{\tau_i}} \) mean of critical defect depth and \( \sigma_{x_{\tau_i}} \) standard deviation of the critical defect depth are estimated by Monte Carlo simulation and defined once for each section of a given thickness and pressure.

**Load-Curve Modeling and Probability of Failure Calculation**

The majority of probabilistic structurally oriented analyses works performed assume a Gaussian distribution, linear or nonlinear, to describe the deterioration process. In recent years, some publi-

\[
\int f_{X_{\tau_i}}(x) \, dx = F_{X_{\tau_i}}(x) = \Phi \left( \frac{Z - \mu_{x_{\tau_i}}}{\sigma_{x_{\tau_i}}} \right) \tag{12}
\]

Where

\[
x_{cr} = \frac{x_{22} - x_{11}}{t_2 - t_1} \tag{4}
\]

\[
\mu_r = \frac{1}{N} \sum_{i=1}^{N} V_i \tag{5}
\]

\[
\sigma_r = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (V_i - \mu_r)^2} \tag{6}
\]

**Gaussian Distribution of Corrosion Growth Rate**

Assuming a linear representation of corrosion process given by

\[
x(\tau) = x_0 + V \tau \tag{7}
\]

Probabilistic load-curve evolution with time is given by the following:

\[
f_{X_{\tau_i}}(x) = N\left( x / \mu_{x_{\tau_i}}(\tau), \sigma_{x_{\tau_i}}(\tau) \right)
\]

\[
= \frac{1}{\sigma_x(\tau) \sqrt{2\pi}} \exp \left( -\frac{(x - \mu_x(\tau))^2}{2\sigma_x^2(\tau)} \right) \tag{9}
\]

\[
\mu_x(\tau) = x_0 + \mu_r \tau \tag{10}
\]

\[
\sigma_x^2(\tau) = \sigma_r^2 \tau^2 \tag{11}
\]

where \( x_0 \) represents corrosion depth reported in the last inspection. The area of failure surface between the resistance curve and the intersection point is reached by integrating the resistance curve

\[
\int f_{X_{\tau_i}}(x) \, dx = F_{X_{\tau_i}}(x) \tag{12}
\]
However, the failure surface included between the intersection point and the load curve is reached integrating the following load curve:

\[
\int_{Z}^{\infty} f_{X(t)}(x) \, dx = F_{X(t)}(x)
\]

\[
= 1 - \Phi \left( \frac{Z - \mu_x(\tau)}{\sigma_x(\tau)} \right)
\]

\[
\Phi \text{ is the cumulative function of the normal distribution.}
\]

\[
P_f = F_{X(t)}(x) + F_{X_s}(x).
\]

\[
\text{Gamma Distribution of Corrosion Rate}
\]

Treatment of data gathered from a pipeline pigged several times indicates that normal distribution of parameters is no longer justified (Fig. 3). The choice of this distribution is supported by its mathematical ease. The corrosion process is an irreversible process and cannot agree with negative values of the corrosion growth rate. Modeling the corrosion rate with a normal distribution proved in certain works that it is truncated with negative values (Desjardins 1993). This truncation, even if it seems to be logical in the sense that negative values do not have a physical meaning, leads to some information loss which can have a negative impact on the estimation efficiency. This disadvantage can be avoided using strictly positive distributions of the gamma family (Younsi and Smati 2005; van Noortwijk et al. 2007). More gamma distribution is versatile and able to take different form of distribution, including normal distribution.

Gamma distribution is given by the following expression:

\[
f(V) = \frac{\beta^n}{\Gamma(n)} V^{n-1} \exp(-V^n) ; \quad V > 0,
\]

with

\[
\alpha = \left( \frac{\mu_x}{\sigma_x} \right)^2
\]

\[
\beta = \frac{\mu_x}{\sigma_x},
\]

where \(\alpha\) = scale parameter and \(\beta\) = form parameter.

On the other hand, the transformations in Eqs. 10 and 11 are valid only for the normal distribution family. For this reason, passing from corrosion-rate distribution toward the load-curve distribution with time becomes an issue.

In this work, to avoid this drawback, transition from corrosion-rate distribution to load-curve distribution with time, assuming a linear corrosion process described in Eq. 7, is made possible using the PTM (Soong 1973; Kadry et al. 2007), and the load curve can then be expressed as follows (Appendix B):

\[
f_{X(t)}(x) = \frac{1}{\tau} \frac{\beta^n}{\Gamma(n)} \left( \frac{x - x_0}{\tau} \right)^{n-1} \exp \left( -\frac{(x - x_0)}{\tau} \right)
\]

To evaluate the probability of failure, this method is similar to the approach described in the previous paragraph. However, the part of the failure surface between the intersection point and the load curve is obtained using a numerical integration. Intersection points \(Z(\tau)\) between the load and resistance curves can be defined resolving the following optimization model:

\[
\left[ f_{X(t)}(x) - F_{X_s}(x) \right]^2 \Rightarrow \text{Min},
\]

TABLE 1—GAS PIPELINE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Pipeline Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Diameter</td>
</tr>
<tr>
<td>Wall thickness</td>
</tr>
<tr>
<td>Steel type</td>
</tr>
<tr>
<td>Minimal stress</td>
</tr>
</tbody>
</table>

TABLE 2—SUMMARY OF INSPECTION RESULTS

<table>
<thead>
<tr>
<th>Inspection tool</th>
<th>Inspection 1</th>
<th>Inspection 2</th>
<th>Number of Matched Defects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of corrosion points found</td>
<td>MFL</td>
<td>MFL</td>
<td>–</td>
</tr>
<tr>
<td>Maximum depth of corrosion defect found mm</td>
<td>4294</td>
<td>5307</td>
<td>940</td>
</tr>
<tr>
<td>Corrosion quality</td>
<td>External</td>
<td>External</td>
<td>–</td>
</tr>
</tbody>
</table>
Fig. 4—Computation diagram.
Probability of Failure Before and After Repairs

An optimal planning of preventive repairs can be established by simulating failure probability before and after repair of every corrosion defect and comparing results to a threshold criterion adopted by the operator. “Threshold value” used in worldwide practice refers to the failure probability by kilometer (DNV RP-F101 2004). In a corroded pipeline, every kilometer may contain many corrosion defects with different sizes. Following reliability theory and assuming that corrosion defects are independent elements installed in a series (Lee and Pyun 2002; Lecchi 2011), the probability of failure at τ of the pipeline section J can be given as follows:

\[ P_J(\tau) = 1 - \prod_{i=1}^{N_J} (1 - P_{\beta i}(\tau)) \]  

where \( P_{\beta i} \) = probability of failure of a corrosion defect i of the kilometre J and \( N_J \) = number of corroded defects within the kilometer J.

It can be concluded from this assumption that the probability of failure of a segment must be determined from probabilities of failure of every corrosion defect detected. Every defect contributes to the calculation of the whole failure probability.

Consequently, a pipeline section containing a high number of defects of average size can be, in certain cases, more dangerous than a section with a smaller number of defects with significant corrosion dimensions. This fact is more often ignored in practice where the analysis is performed using the known standards and is performed defect per defect.

Inspections are followed by a preventive intervention program. This program can be established using the estimated probability of failure. This value is determined by applying the relation in Eq. 21 and allocating a probability of zero for every corrosion defect for repair. This reduces \( N_J \) by a quantity equivalent to a number of repaired defects. Combination of the model in Eq. 18 and the relation in Eq. 21 allows the elaboration of an experimentation stand where the influence of actions, like repairs and segment replacement, on the probability of failure estimation can be simulated beforehand. In the same way, an optimal planning of the future inspection date can be performed.

Case Study: Algerian Natural Gas pipeline

The following application concerns a segment of 80 km of an Algerian natural gas pipeline, with the illustrated characteristics in Table 1. The facility was pigged two times with 5-year intervals (Table 2).

To take into account the spatial variability, the whole pipeline is divided into sections, and defects are grouped into classes according to their depths. Probability of failure estimation for each value \( \tau_i \) is then obtained following several steps (Fig. 4).

Fig. 5 represents the resistance curves associated to two different operating pressures (corresponding to two different locations of the gas pipeline) obtained using Eq. 3 and the model described in Appendix A.

Results of corrosion growth-rate modeling using normal and gamma distributions are represented in Fig. 6, and subsequent load curves are estimated using the relations in Eq. 9 that we assign as “NV model” and Eq. 18 that we assign as “GV model,” respectively, in Figs. 7 and 8.

Fig. 9 represents, for two different initial corrosion depths, the probability density function after 5 years, estimated using the NV and GV models. The degree of agreement with the histogram obtained from the real data after two inspections is quantified by \( \chi^2 \) criteria. These results show the best-quality estimation of the GV model.

Fig. 10 represents the failure probability evolution with time on the basis of the NV and GV models. Two values of operating pressure and different initial depths are taken into account. The following can be concluded from these results that:

- For small initial corrosion depths, characterized by a low probability of failure, the NV model overestimates the failure probability for a short time horizon after inspection (1 to 2 years) and underestimates it for long intervals (4 to 5 years). In addition, the more the operating pressure increases, the more the intersection points between NV and GV curves tend to move to the right side.
- For significant initial corrosion depths, characterized by high probabilities of failure, the NV model overestimates the failure probability for a short time horizon after inspection and gives similar results as the GV model for long periods.

In practice, post-inspection repairs are carried out just after the inspection and concern defects with significant depth of the metal. Thus, from the previous conclusions, the use of NV models leads to many unnecessary repairs and increases significantly maintenance costs.

For the whole gas pipeline, treatment of inspection data is performed using three approaches (Table 3):
• The ASME B31G approach (1991) (Appendix A)
• The reliability structural approach, based on the NV model.
• The approach developed in this paper, based on the GV model.

The results are summarised in Table 3.

Figs 11 and 12 represent, respectively, the probability of failure evolution with time of a segment estimated with the NV and GV models. Repairs were assumed to be performed every year on the segment with a probability of failure over $10^{-3}$ before repairs.

Conclusions
Pipeline inspection results using smart pigs provide a static picture characterizing the deterioration state of the pipeline at the moment of inspection. Because of the dynamic and the random nature of pipeline corrosion, these results can be represented by a stochastic process related to time. In this context, using Gaussian assumption leads to an overestimation of the failure probability translated into practice by unnecessary repairs. To take into account the irreversible nature of the deterioration phenomenon, their representation using a positive process, the gamma type, is fitted to this purpose.

The model developed in this paper authorizes a more rigorous estimation of probabilities of failure, reduces maintenance costs, and permits a quantitative evaluation of the failure risk between two inspections and the optimal planning of the preventive maintenance programs to assure the low-risk level to be practical.

Nomenclature

$D$ = outer diameter of the pipeline  
$GV$ = gamma model  
$L$ = defect length  
$M$ = Foliás factor  
$N_j$ = number of corroded defects within the kilometer $J$  
$NV$ = normal model  
$P_{fi}$ = probability of failure of a corrosion defect $i$ of the kilometer $J$  
$S_{\text{min}}$ = specified minimum yielding stress  
$t$ = wall thickness  
$x$ = defect depth  
$\alpha$ = scale parameter  
$\beta$ = form parameter  
$\mu_V$ = mean of corrosion growth rate  
$\mu_{X_{cr}}$ = mean of critical defect depths  
$\sigma_V$ = standard deviation of corrosion growth rate

Fig. 7—NV load-curve evolution with time ($X_0 = 66\%$ of the wall thickness).

Fig. 8—GV load-curve evolution with time ($X_0 = 66\%$ of the wall thickness).

Fig. 9—Histogram and probabilistic distributions of corrosion depth after 5 years for two classes of initial depth (29 and 63%).
\( \sigma_{x_c} \) = standard deviation of the critical defect depth

\( \Phi \) = is the cumulative function of the normal distribution

References


Appendix A—Model Based on the ASME B31 G Code

Failure stress $S_f$ can be represented by ASME B31 G (1991) as follows:

$$S_f = 1.1S_{\min} \left[ \frac{1 - 2x}{3t} \right]^{31}, \quad \text{..(A-1)}$$

where

$$M = \sqrt{1 + 0.8 \left( \frac{L}{\sqrt{Dt}} \right)^2}, \quad \text{..(A-2)}$$

The critical corrosion depth can then be expressed by

$$x_{cr} = \begin{cases} 
3 \left( \frac{1.1S_{\min} - S_f}{1.1S_{\min} - M^{-1}S_f} \right)t & \text{if } \sqrt{0.8 \left( \frac{L}{\sqrt{Dt}} \right)^2} \leq 4 \\
1 - \frac{S_f}{1.1S_{\min}}t & \text{if } \sqrt{0.8 \left( \frac{L}{\sqrt{Dt}} \right)^2} > 4
\end{cases}, \quad \text{..(A-3)}$$

where $S_{\min}$=specified minimum yielding stress, $M$=Folias factor, $x$=defect depth, $L$=defect length, $D$=outer diameter of the pipeline, and $t$=wall thickness.

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Appendix B—Load-Curve Model Based on Gamma Distribution of Corrosion Growth Rate and PTM

In a general case using the PTM, the density of probability $f_X(x)$ of a random variable $X$, related to another random and continuing variable $V$ with a known density of probability $f_V(v)$, can be obtained applying the following relation:

$$ f_X(x) = f_V\left(g^{-1}(x)\right) \frac{d}{dx} g^{-1}(x) \quad \text{..................................(B-1)} $$

Assuming a linear representation of the corrosion process given by Eq. 7, we obtain

$$ v = g^{-1}(x) = \frac{x-x_0}{\tau} \quad \text{..................................(B-2)} $$

and

$$ \frac{dv}{dx} = \frac{d}{dx} g^{-1}(x) = \frac{1}{\tau} \quad \text{..................................(B-3)} $$

From these relations, we can express the load-curve evolution with time, assuming gamma distribution of corrosion growth rate as follows:

$$ f_X(x) = \frac{1}{\tau} \beta^\alpha \frac{x-x_0}{\tau}^{\alpha-1} \exp\left(-\frac{x-x_0}{\tau}\right) \quad \text{..................................(B-4)} $$

or

$$ f_X(x) = \frac{1}{\Gamma(\alpha)} \left\{ \frac{x-x_0}{\tau} \right\}^\alpha \exp\left(-\frac{x-x_0}{\tau}\right) \quad \text{....................(B-5)} $$

### Table 3—Number of Repairs Per Year Using Different Approaches

<table>
<thead>
<tr>
<th>Methodology</th>
<th>1st year</th>
<th>2nd year</th>
<th>3rd year</th>
<th>4th year</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>B 31 G code</td>
<td>116</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>NV model</td>
<td>194</td>
<td>35</td>
<td>242</td>
<td>222</td>
<td>693</td>
</tr>
<tr>
<td>GV model</td>
<td>36</td>
<td>90</td>
<td>74</td>
<td>262</td>
<td>462</td>
</tr>
</tbody>
</table>

Fig. 11—Failure probability per km before and after repairs (1 year after inspection).
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