**Summary**

Gas transient flow in a gas pipeline and gas tank is critical in flow assurance. Not only does leak detection require a delicate model to simulate the complicated yet dramatically changed phenomena, but gas-pipeline and gas-tank design in metering, gathering, and transportation systems demands an accurate analysis of gas transient flow, through which efficient, cost-effective operation can be achieved.

Traditionally, there are two types of approaches used to investigate gas transient flow: One involves treating gas as ideal gas so that the ideal-gas law can be applied, and the other considers gas as real gas, allowing the gas-compressibility factor to come into play. Needless to say, the former method can result in an analytical solution to gas transient flow with a deviation from the real-gas performance, which is very crucial in daily operation. The latter approach requires a numerical method to solve the governing equation, leading to instability issues with a more-accurate result. Our literature review indicated that no study considering the effect of changing gas viscosity on the transient flow was available; therefore, this effect was included in our study.

Our investigation showed that viscosity does have a significant influence on gas transient flow in pipe- and tank-leakage evaluation. In this study, a comprehensive evaluation of all variables was performed to determine the most-important factors in the gas transient flow. Several case studies were used to illustrate the significance of this study. Engineers can perform a more-reliable evaluation of gas transient flow by following the method used in our study.

**Introduction**

The importance of gas transient flow in a gas pipeline and gas tank cannot be overemphasized in flow assurance. Owing to the advancement of technology in drilling, completion, offshore operation, and long-distance pipeline transportation, more and more offshore gas fields are being developed by means of subsea wellheads, with production being transported by long subsea pipelines, which increases gas-project feasibility through cost reduction because no platform is required. Yet, this type of development setup brings complexity and makes operation and maintenance challenging. Without the support of a platform, many operations, such as pipeline-leakage detection, pipeline pigging, pipeline testing, well shut-in, production startup or restart, and well workover, are difficult to conduct. Sometimes, it is impossible to fulfill the operation. Under such conditions, gas-transient-flow analysis stands out as a vital approach in diagnostics, and an accurate analysis of gas transient flow, through which efficient, cost-effective operation can be achieved.

Both analytical and numerical methods are applied in gas transient-flow analysis. Needless to say, the former method can result in a fast and stable solution to gas transient flow, but it can be applied only in a very simple or ideal condition. Ideal gas is always assumed to obtain an analytical solution. In the field, we face the real gas with complex pipeline networks. To deal with these real situations, we resort to the numerical method, through which more-accurate results can be achieved.

Many researchers have conducted investigations on gas transient flow in gas pipelines and tanks. Luongo (1986) studied the gas transient flow in a constant-cross-section pipe. He linearized the partial-differential equation and developed a numerical solution to the linear parabolic partial-differential equation. In his derivation, friction factor was calculated from steady-state condition or, in other words, constant friction factor for transient flow. Zhou and Adewumi (2000) included the kinetic-energy term in the governing equation and solved the partial-differential equation numerically. Reddy et al. (2006) built dynamic simulation models using a fully nonlinear second-order-accurate finite-difference method for state estimation and leak detection. Scott and Satterwhite (1998) evaluated blockage detection with a backpressure technique, assuming “fully rough flow.” The meaning of fully rough flow is constant friction factor.

**Gas-Flow Regime**

When gas passes through a restriction (e.g., a throttled pipe, a nozzle, an orifice, or a valve in a pipe) and into a lower-pressure environment, it experiences a choked flow. The knowledge of gas flow through restrictions will help us understand the gas transient flow in pipe and tank. Gas flow out of pipeline or tank can be divided into subsonic flow and sonic flow, according to flow regime. Sonic flow is defined as the point at which the fluid-flow velocity through a choke or throttled pipe reaches the velocity of sound in the fluid under the in-situ condition. In other words, the upstream cannot “feel” the pressure wave propagated from downstream upward because the fluid is travelling in the opposite direction with the same velocity under sonic-flow conditions. An easy way to determine if the flow falls into the sonic-flow region is the critical pressure ratio. Sonic flow occurs when the downstream/upstream pressure ratio is equal to or less than the critical pressure ratio, which is expressed as

\[
\frac{p_{\text{down}}}{p_{\text{up}}} = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}, \quad \text{.......................................................... (1)}
\]

where \( p_{\text{down}} \) is the downstream pressure, \( p_{\text{up}} \) is the upstream pressure, \( k = C_p/C_v \) is the specific-heat ratio of fluid, \( C_p \) is the fluid heat capacity at constant pressure, and \( C_v \) is the fluid heat capacity at constant volume.

From its name, we know that subsonic flow exists when flow velocity is less than the sound velocity in the fluid at the in-situ condi-
tion. For subsonic flow, the pressure ratio is greater than the critical pressure ratio.

Methodology
Gas transient flow often occurs in gas flowing through a choke, a throttled pipe, a gas-tank discharge, and/or a leaking pipe or tank. The material-balance principle is applied in the gas transient flow. Considering a section of a pipeline or a tank that has an inlet and an outlet, the moles of gas that accumulate in that pipeline or tank can be obtained by

\[ n_{\text{accum}} = n_i + n_{\text{in}} - n_{\text{out}} \]  

(2)

where \( n_{\text{accum}} \) is the moles of gas accumulated in the pipeline or the tank, \( n_i \) is the initial moles of gas in the pipeline or the tank, \( n_{\text{in}} \) is the moles of gas that flow into the pipeline or the tank, and \( n_{\text{out}} \) is the moles of gas that flow out of the pipeline or the tank.

Gas flow into the pipeline or the tank is subject to less pressure change. The estimation of gas-volume flow into the pipeline or the tank is easy and reliable. Gas that flows out of the pipeline or the tank through a choke, a throttled pipe, a gas-tank discharge, and/or a leaking pipe or tank experiences a dramatic pressure change. Gas flow-out volume varies quickly as pipeline or tank pressure changes. The calculation of flow-out volume is complicated and difficult to predict. The accurate estimation of flow-out volume is of significance in gas-tank discharge and pipeline- and tank-leakage detection. Therefore, in this paper, we assume that the flow-in gas volume is known or given. The flow-out volume calculation is our objective. The gas-tank discharge time is also important in flow assurance.

From the practical-application point of view, we discuss two important gas-transportation phenomena: (1) subsonic flow of gas through a throttled pipe in the case of dramatic pipe-diameter change; and (2) gas discharge from the pipeline/tank or the pipeline/tank leakage, which can be simulated as choke performance. The real-gas law gives the pressure/volume/temperature relationship of gas at initial condition:

\[ p_i V_i = z_i n_i R T_i \]  

(3)

where \( p_i \) is the initial gas pressure in the pipeline or the tank, \( V_i \) is the gas volume at initial condition, \( z_i \) is the gas compressibility at initial condition, \( n_i \) is the number of moles, \( R \) is the universal gas constant, and \( T_i \) is the initial temperature in the pipeline or the tank.

When pressure declines because of gas flow out of the pipeline or the tank, Eq. 3 becomes

\[ p V = z n R T \]  

(4)

where \( p \) is the gas pressure in the pipeline or the tank, \( V \) is the gas volume at pressure \( p \) and temperature \( T \), \( z \) is the gas compressibility at pressure \( p \) and temperature \( T \), \( n \) is the number of moles, \( n=n_i \) for constant composition expansion, and \( T \) is the temperature in the pipeline or the tank.

If there is no gas flow into the pipeline or tank \( (n_{\text{in}}=0) \), the gas flow out of the pipeline or tank is a constant-composition-expansion process. Eq. 1 can be expressed by

\[ \frac{p}{z} = \frac{p_i}{z_i} \left( 1 - \frac{V_{\text{out}}}{V_i} \right) \]  

(5)

where \( V_{\text{out}} \) is the gas-volume flow out of the pipeline or the tank and \( V_i \) is the initial gas volume.

Gas Flow Through a Throated Pipe. Now, considering the case of subsonic flow of gas through a throttled pipe, gas-volume flow through the pipe can be calculated by the product of flow rate and time. If the pipe is in the horizontal direction, the flow rate can be calculated from the pressure drop in the throttled pipe, or the Weymouth (1912) equation, which is

\[ q = \frac{C_T}{P_{\text{sc}}} \left[ \left( \frac{p_{\text{up}}^2 - p_{\text{down}}^2}{p_{\text{sc}}^2} \right)^2 \right]^{1/2} \gamma_T (z_T^2 + f L) \]  

(6)

where \( q \) is the gas-flow rate, \( C_T \) is the constant for unit conversion, \( D \) is the throttled-pipe diameter, \( T_s \) is the standard-condition temperature, \( P_{\text{sc}} \) is the standard-condition pressure, \( z_T \) is the average gas compressibility equal to \( (z_{\text{up}} + z_{\text{down}})/2 \), \( T \) is the average temperature equal to \( T_{\text{up}} + T_{\text{down}}/2 \), \( \gamma_T \) is the gas specific gravity, \( L \) is the throttled-pipe length, and \( f \) is the friction factor, which can be calculated by the Jain (1976) correlation,

\[ \frac{1}{\sqrt{f}} = 1.14 - 2 \log \left( \frac{e_D}{D} + \frac{21.25}{N_{\text{Re}}} \right) \]  

(7)

where \( e_D \) is the relative roughness, which is defined as the ratio of the absolute roughness to the pipe internal diameter,

\[ e_D = \frac{e}{D} \]  

(8)

and \( N_{\text{Re}} \) is the Reynolds number, which can be expressed as a dimensionless group,

\[ N_{\text{Re}} = \frac{D u \rho}{\mu} \]  

(9)

where \( \epsilon \) is the absolute roughness, \( \mu \) is the gas viscosity, \( \rho \) is the gas density, and \( u \) is the gas-flow velocity.

Substituting Eq. 9 into Eq. 7 and then substituting Eq. 7 into Eq. 6, we obtain

\[ q = \frac{C_T}{P_{\text{sc}}} \left[ \left( \frac{p_{\text{up}}^2 - p_{\text{down}}^2}{p_{\text{sc}}^2} \right)^2 \right]^{1/2} \gamma_T (z_T^2 + f L) \]  

(10)

Because gas-flow velocity can be expressed as

\[ q = \frac{1}{4} D^2 u \]  

(11)

Eq. 10 can be rearranged into

\[ q = \frac{C_T}{P_{\text{sc}}} \left[ \left( \frac{p_{\text{up}}^2 - p_{\text{down}}^2}{p_{\text{sc}}^2} \right)^2 \right]^{1/2} \gamma_T (z_T^2 + f L) \]  

(12)

Obviously, Eq. 12 is an implicit equation for flow rate \( q \). Average gas compressibility, density, and viscosity are functions of pressure, temperature, and composition. Because pressure and temperature change with time, so do \( z \)-factor, density, and viscosity; the analytical solution to the flow rate is impossible, and Eqs. 5 and 12 need to be solved simultaneously. The procedure comprises the calculation of \( z \)-factor, density, and viscosity at updated pressure and temperature. A small timestep needs to be applied to solve the equations numerically.

Gas Discharge From the Pipeline/Tank or Pipeline/Tank Leakage. Gas discharge from the pipeline/tank or pipeline/tank leakage can be treated as choke performance. As mentioned in
the preceding gas-flow regime, at the early stage of gas discharge from the pipeline/tank or pipeline/tank leakage, sonic flow dominates because of the high upstream pressure compared with the low downstream pressure. Then, it changes to subsonic flow as the pipeline or tank pressure reduces. Under the sonic-flow condition, gas velocity reaches its maximum value. The flow rate of sonic flow can be calculated by

\[
q = 879 C_p A_p \left( \frac{k}{\gamma g T_w} \right)^{\frac{2}{k+1}} \left( \frac{2}{k+1} \right)^{k+1} \left( \frac{d_1}{d_2} \right)^{k+1} \left( \frac{d_2}{d_1} \right)^{k+1}, \tag{13}
\]

where \(A\) is the cross-sectional area of the choke, \(T_w\) is the upstream temperature, and \(C_p\) is the choke-discharge coefficient, which can be determined on the basis of Reynolds number and choke-/pipe-diameter ratio. The Guo and Ghalambor (2005) correlation provides a feasible way to estimate \(C_p\):

\[
C_p = \frac{d_1}{d_2} + 0.367 \left( \frac{d_2}{d_1} \right)^{0.25} \log \left( N_k \right) - 4 \tag{14}
\]

where \(d_1\) is the pipe or tank diameter and \(d_2\) is the choke diameter.

One misconception of gas-flow rate under the sonic-flow regime should be avoided. On the basis of the fact that gas velocity reaches its maximum value, some may draw the conclusion that gas-flow rate does not change even as upstream pressure increases. This is not true. As can be seen from Eq. 13, gas-flow rate increases as upstream pressure increases. Even though velocity does not change, the density increases as upstream pressure increases. Thus, the mass-flow rate increases. Consequently, volume-flow rate increases. From Eq. 13, we see that an increase in downstream pressure will not affect the gas-flow rate until the ratio of downstream pressure to the ratio of upstream pressure reaches the critical pressure ratio and sonic flow transitions to subsonic flow.

Subsonic flow occurs when the downstream-/upstream-pressure ratio is larger than the critical pressure ratio. Under subsonic-flow condition, the change in the downstream pressure can be “felt” by the upstream pressure. Gas-flow rate can be calculated by

\[
q = 1.248 C_p A_p \left( \frac{2k}{(k-1) \gamma g T_w} \right)^{\frac{2}{k+1}} \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)^\frac{k}{k+1} \tag{15}
\]

Again, both sonic flow and subsonic flow have a choke-discharge coefficient in the flow-rate calculation. As shown by Eq. 14, the choke-discharge coefficient is a function of the choke diameter, the pipe/tank diameter, and the Reynolds number. If we recall the definition of the Reynolds number in Eq. 9, we find that neither Eq. 13 nor Eq. 15 is the explicit equation for gas-flow rate. We face the same difficulty in calculating the gas discharge through a throttled pipe. Because the gas density and viscosity change as upstream and downstream pressure and temperature change, the analytical solution to obtain flow rate as a function of time, pressure, temperature, pipe/tank and choke geometry, and gas composition is impossible. Eqs. 5, 13, and 15 need to be solved simultaneously, considering that gas discharge from the pipeline/tank or pipeline/tank leakage consists of sonic flow and subsonic flow in most cases. Therefore, methods to obtain the z-factor, density, and viscosity are necessary in solving these equations numerically. The Brill and Beggs (1974) correlation provides an accurate way to calculate z-factor:

\[
z = A + 1 - A e^{-z} + CP_{\text{pr}} \tag{16}
\]

where

\[
A = 1.39 (T_n - 0.92)^{0.5} - 0.36 T_n - 0.1,
\]

\[
B = (0.62 - 0.23 T_n) P_{\text{pr}} + \frac{0.066}{T_n - 0.86} - 0.037 P_{\text{pr}}^2 + 0.32 P_{\text{pr}}^3 + 10^7,
\]

\[
C = 0.132 - 0.32 \log (T_n),
\]

\[
D = 10^6,
\]

\[
E = 9 (T_{\text{pr}} - 1),
\]

\[
F = 0.3106 - 0.497 T_n + 0.1824 T_n^2,
\]

and 11.

Once the gas-compressibility factor is provided, gas density can be calculated by

\[
\rho = 2.7 \frac{M_w - P}{28.96 T} \tag{17}
\]

With a given z-factor and density, gas viscosity can be estimated by the Gonzalez et al. (1970) correlation, as

\[
\mu = 10^{-7} K \exp \left( X P \right) \tag{18}
\]

where

\[
K = \frac{9.379 + 0.01607 M_w^2 T^{1.5}}{209.2 + 19.26 M_w + T},
\]

\[
X = 3.448 + \frac{986 M_w}{T} + 0.01009 M_w,
\]

\[
Y = 2.447 - 0.2224 X, \text{ and } M_w \text{ is the molecular weight.}
\]

**Calculation Procedure**

1. Calculate the initial gas volume in the pipe or the tank. Calculate the gas volume at standard condition, which will be used as initial gas volume in Eq. 5.
2. Calculate the gas z-factor, density, and viscosity under initial condition with Eqs. 16, 17, and 18, respectively.
3. Choose a small timestep, and calculate the gas-flow rate out of the pipe or the tank by using Eq. 12, Eq. 13, or Eq. 15, whichever is appropriate. At this stage, initial condition is applied.
4. Calculate the gas-volume flow out of the pipe or the tank using flow time and the gas-flow rate from Step 3.
5. Calculate the pressure and temperature after depleting a certain gas volume in Step 3 by using Eq. 5.
6. Calculate the remaining gas volume at standard condition.
7. Calculate the gas z-factor, density, and viscosity under updated pressure and temperature condition with Eqs. 16, 17, and 18, respectively.
8. Choose a small timestep, and calculate the gas-flow rate out of the pipe or the tank with Eq. 12, Eq. 13, or Eq. 15, whichever is appropriate. At this stage, the updated pressure and temperature condition obtained in Step 5 and the z-factor, density, and viscosity obtained in Step 7 are applied.
9. Calculate the gas-volume flow out of the pipe or the tank using flow time and the gas-flow rate from Step 8.
10. Calculate the pressure and temperature after depleting a certain gas volume in Step 9 by using Eq. 5. At this stage, the gas volume obtained in Step 6 is used as the initial gas volume.
11. Calculate the remaining gas volume at standard condition after the second depletion.
12. Repeat Steps 7 through 11 until the pipe or the tank pressure reaches downstream pressure.

13. Build the relationship between flow rate and time using the data set obtained from the preceding calculation.

Case Study

An example is used to illustrate the application of the numerical method in solving the drainage of a closed pipe filled with gas. We can expect that the gas flow from a closed pipe shows the characteristics of transient flow. The following assumptions are applied for the calculation.

1. Gas in a pipeline is discharged through a throated pipe with a shorter length and smaller diameter than a major pipeline.
2. Single gas phase is discharged to the standard condition (i.e., pressure of 14.7 psia and temperature of 520°R).
3. Upstream and downstream temperatures are constant.
4. Frictional pressure drop in the throated pipe dominates. Frictional pressure drop in the major pipeline can be ignored.

Input data for the flow-rate-vs.-time analysis are listed in Table 1. Table 2 shows the calculated parameters, flow rate, and pipe pressure as functions of time. The flow-rate-vs.-time profile is shown in Fig. 1. Fig. 2 illustrates the plot.

### Table 1—Input Data for the Calculation of Gas-Flow Rate vs. Time for Gas Discharges From a Closed Pipe

<table>
<thead>
<tr>
<th>Pipe Length</th>
<th>10,000 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe Diameter</td>
<td>1 ft</td>
</tr>
<tr>
<td>Initial Pressure</td>
<td>5,000 psi</td>
</tr>
<tr>
<td>Initial Temperature</td>
<td>560°R</td>
</tr>
<tr>
<td>Downstream Pressure</td>
<td>14.7 psi</td>
</tr>
<tr>
<td>Downstream Temperature</td>
<td>520°R</td>
</tr>
<tr>
<td>Gas-Specific Gravity</td>
<td>0.7</td>
</tr>
<tr>
<td>Throated-Pipe Length</td>
<td>100 ft</td>
</tr>
<tr>
<td>Throated-Pipe Diameter</td>
<td>0.2 ft</td>
</tr>
<tr>
<td>Pressure at Standard Condition</td>
<td>14.7 psi</td>
</tr>
<tr>
<td>Temperature at Standard Condition</td>
<td>520°R</td>
</tr>
<tr>
<td>Absolute Roughness of Pipe</td>
<td>0.0006 in.</td>
</tr>
</tbody>
</table>

### Table 2—The Calculated Parameters, Flow Rate, and Pipe Pressure vs. Time

<table>
<thead>
<tr>
<th>Discharge Time, seconds</th>
<th>Pipe Pressure, psia</th>
<th>( q ), MMscf/d</th>
<th>Remaining Gas in Pipe, MMscf</th>
<th>Gas Density, lb/cu-ft</th>
<th>Viscosity, cp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,012</td>
<td>0.9495</td>
<td>1,277.3</td>
<td>0.015</td>
<td>26.186</td>
</tr>
<tr>
<td>2</td>
<td>5,008</td>
<td>0.9490</td>
<td>1,276.6</td>
<td>0.021</td>
<td>26.165</td>
</tr>
<tr>
<td>4</td>
<td>5,002</td>
<td>0.9485</td>
<td>1,275.6</td>
<td>0.029</td>
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<tr>
<td>7</td>
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<td>1,274.1</td>
<td>0.040</td>
<td>26.096</td>
</tr>
<tr>
<td>11</td>
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<td>0.9465</td>
<td>1,272.1</td>
<td>0.057</td>
<td>26.039</td>
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<tr>
<td>16</td>
<td>4,969</td>
<td>0.9449</td>
<td>1,269.4</td>
<td>0.079</td>
<td>25.960</td>
</tr>
<tr>
<td>24</td>
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<td>0.9427</td>
<td>1,265.5</td>
<td>0.110</td>
<td>25.850</td>
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<tr>
<td>34</td>
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<td>0.9396</td>
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<td>99</td>
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<td>0.566</td>
<td>24.208</td>
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<td>196</td>
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<tr>
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<td>0.8747</td>
<td>1,141.6</td>
<td>1.049</td>
<td>22.386</td>
</tr>
<tr>
<td>386</td>
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<td>0.8485</td>
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<td>20.983</td>
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<tr>
<td>542</td>
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<td>19.144</td>
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<tr>
<td>760</td>
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<td>0.7816</td>
<td>931.5</td>
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<td>16.796</td>
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<tr>
<td>1,065</td>
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<td>2.882</td>
<td>13.914</td>
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<td>1,492</td>
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<td>0.7529</td>
<td>675.9</td>
<td>3.340</td>
<td>10.574</td>
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<tr>
<td>2,089</td>
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<td>0.8013</td>
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<td>3.547</td>
<td>7.027</td>
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<tr>
<td>2,926</td>
<td>715</td>
<td>0.8882</td>
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<td>3.286</td>
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<tr>
<td>4,097</td>
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<td>5,497</td>
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<td>6,997</td>
<td>20</td>
<td>0.9975</td>
<td>13.7</td>
<td>0.238</td>
<td>0.112</td>
</tr>
</tbody>
</table>
of pipe pressure vs. discharge time. Table 2 and Figs. 1 and 2 show that flow rate and pressure decline quickly at the early stage. The decline rate slows down at late time.

**Sensitivity Analysis**

For convenience, the sensitivity analysis uses the same example as in case study.


<table>
<thead>
<tr>
<th>Table 3—The Effect of Viscosity on the Flow-Rate-Vs.-Time Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge Time, seconds</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>4</td>
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<th>Table 4—The Effect of Viscosity on the Pipe-Pressure-Vs.-Time Profile</th>
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<tr>
<td>Discharge Time, seconds</td>
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Sensitivity of Gas Viscosity. To evaluate the effect of the gas viscosity on the flow-rate-vs.-time and pressure-vs.-time profiles, two cases are compared with one another. One case is the changing viscosity with pressure decline during the discharge. Another case is constant viscosity throughout the entire discharge period. All other input data are the same for these two cases. Table 3 compares the flow rate vs. time for both cases.

Table 3 indicates that differences in the flow-rate result from viscosity increase as pipe pressure declines. Viscosity has a significant impact at the late period. The difference in flow rate can be up to 20%. Table 4 compares the pipe pressure vs. time for both cases. It is obvious that differences in pipe pressure result from viscosity increase as pipe pressure declines. This follows the same trend as that in the gas-flow rate. Again, viscosity has a significant impact at the late period. The difference in pressure can be up to 60%.

Sensitivity of Pipe Volume. The effect of pipe volume on flow-rate-vs.-time and pressure-vs.-time profiles is evaluated by comparing a throated-pipe length of 100 ft with a length of 120 ft. The plots of flow rate vs. time and pressure vs. time are shown in Figs. 5 and 6.

Clearly, Figs. 5 and 6 depict large differences in flow rate and pipe pressure during the middle period of discharge. But they are different from the uncertainty of pipe volume. The flow-rate-vs.-time curves cross each other in Fig. 5, while in Fig. 3, they do not. Therefore, we can differentiate pipe-volume uncertainty from throated-pipe-diameter uncertainty. Figs. 5 and 3 can serve as “type curves” in the uncertainty diagnostic.

Sensitivity of the Throated-Pipe Diameter. The effect of the throated-pipe diameter on the flow-rate-vs.-time and pressure-vs.-time profiles is evaluated by comparing a throated-pipe diameter of 0.2 ft with a diameter of 0.22 ft. The plots of flow rate vs. time and pressure vs. time are shown in Figs. 5 and 6.

Similarly, Figs. 7 and 8 depict large differences in flow rate and pipe pressure during the middle period of discharge. They are similar to the throated-pipe-diameter differences because of the uncertainty of the throated-pipe diameter. The flow-rate-vs.-time curves cross each other in Figs. 5 and 7. The pipe-pressure-vs.-time curves are similar to each other in Figs. 6 and 8. Therefore, we cannot differentiate throated-pipe-diameter uncertainty from throated-pipe-length uncertainty on the basis of flow-rate-vs.-time and pressure-vs.-time profiles only. Comparing the input data with the results of sensitivity analysis of throated-pipe diameter and throated-pipe length, we found that flow rate vs. time and pressure vs. time are more sensitive to diameter than to length. Figs. 9 and 10 show that a 10% increase in...
Flow rate vs. time

![Flow rate vs. time graph](image)

**Fig. 3**—Comparison of flow-rate-vs.-time profiles for two cases with different pipe volumes.

Pipe pressure vs. time

![Pipe pressure vs. time graph](image)

**Fig. 4**—Comparison of pipe-pressure-vs.-time profiles for two cases with different pipe volumes.
Fig. 5—Comparison of flow-rate-vs.-time profiles for two cases with different pipe diameters.

Fig. 6—Comparison of pipe-pressure-vs.-time profiles for two cases with different pipe diameters.
Fig. 7—Comparison of flow-rate-vs.-time profiles for two cases with different pipe lengths.

Fig. 8—Comparison of pipe pressure vs. time profiles for two cases with different pipe lengths.
the length results in less flow-rate-vs.-time and pressure-vs.-time changes than in the case of a 10% increase in the diameter. Therefore, we can conclude that pipe diameter is the most important element in gas flow.

Conclusions

The following conclusions can be drawn upon the completion of this study:

This investigation provided a numerical method to evaluate the gas transient flow in throated pipe, closed pipe and/or tank discharge, and pipe/tank leakage. It is a practical way to perform quick calculation in the field.

1. Both flow rate and pressure decline quickly at the early stage. Then, the declines slow down at the late period.
2. Gas viscosity has an important effect on flow rate and pipe pressure at the late period.
3. Pipe volume has a significant impact on the flow-rate-vs.-time and pressure-vs.-time profiles. Pipe-volume uncertainty can be differentiated from throated-pipe length or diameter by “type curves” in flow-rate-vs.-time plots.
4. Qualitatively, throated-pipe-length uncertainty cannot be differentiated from throated-pipe-diameter uncertainty if we depend on flow-rate-vs.-time and pressure-vs.-time “type curves” only.
5. It is possible to differentiate throated-pipe-length uncertainty from throated-pipe-diameter uncertainty using a quantitative method because flow rate and pressure are more sensitive to diameter.
6. Pipe or choke diameter is the most important variable in transient-gas-flow analysis.

Nomenclature

\( A = \) cross-sectional area of choke
\( C = \) constant for unit conversion
\( C_D = \) choke-discharge coefficient
\( C_{p,0} = \) fluid heat capacity at constant pressure
\( C_{v,0} = \) fluid heat capacity at constant volume
\( d_1 = \) pipe or tank diameter
\( d_2 = \) choke diameter
\( D = \) throated-pipe diameter
\( e_D = \) relative roughness
\( f = \) friction factor
\( k = \frac{C_p}{C_v} = \) specific-heat ratio of fluid
\( L = \) throated-pipe length
\( M_w = \) molecular weight
\( n = \) number of moles
\( n_{\text{accum}} = \) moles of gas accumulated in the pipeline or the tank
\( n_i = \) initial moles of gas in the pipeline or the tank
\( n_{in} = \) moles of gas that flow into the pipeline or the tank
\( n_{out} = \) moles of gas that flow out of the pipeline or the tank
\( N_{Re} = \) Reynolds number
\( p = \) gas pressure in the pipeline or the tank
\( p_{\text{down}} = \) downstream pressure
\( p_i = \) initial gas pressure in the pipeline or the tank
\( P_{pr} = \) pseudoreduced pressure
\( p_{sc} = \) standard-condition pressure
\( p_{up} = \) upstream pressure
\( q = \) gas-flow rate
\( R = \) universal gas constant
\( T = \) temperature in the pipeline or the tank
\( T = \) average temperature
\( T_i = \) initial temperature in the pipeline or the tank
\( T_{pr} = \) pseudoreduced temperature
\( T_{sc} = \) standard-condition temperature
\( T_{up} = \) upstream temperature
\( u = \) gas-flow velocity
\( V = \) gas volume
\( V_i = \) initial gas volume
\( V_{out} = \) gas-volume flow out of the pipeline or the tank
\( z = \) gas compressibility
\( \bar{z} = \) average gas compressibility
\( z_i = \) gas compressibility at initial condition
\( \gamma_g = \) gas specific gravity
\( \varepsilon = \) absolute roughness
\( \mu = \) gas viscosity
\( \rho = \) gas density

Figure 9—Comparison of the sensitivities of throated-pipe length and throated-pipe diameter on a flow-rate-vs.-time profile.
Fig. 10—Comparison of the sensitivities of throated-pipe length and throated-pipe diameter on a pipe-pressure-vs.-time profile.

References

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